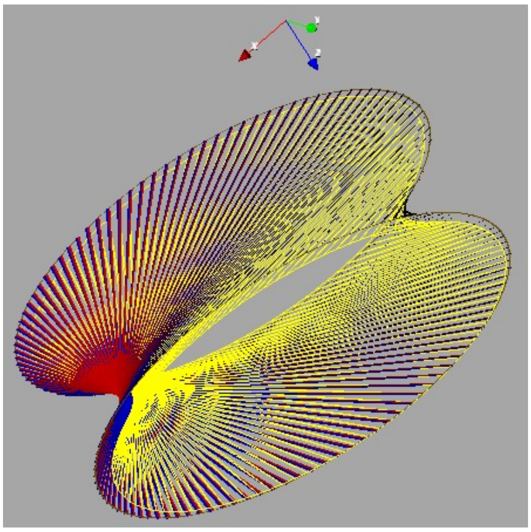
Analysis of tensegrity structures with XC



XC is a finite element program oriented to civil engineering. It is conceived as Open Source Software since we are developing it on the strong foundations of *OpenSees* and making heavy use of other OSS like Python, VTK and CGAL.
This case study deals with the analysis of several 2D and 3D tensegrity units.
In all cases, the results issued by the finite element model in XC compares very well with those forthcoming from symbolic analysis.
A 3D-corotational truss formulation with XC appears to be an effective technique for solving the geometrically nonlinear problem that arises in a tensegrity structure.

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Introduction

Tensegrity structures are usually defined as structural systems that maintain their shape by using a discontinuous set of compressive elements (struts) that are connected to a continuous net of prestressed tensile elements (cables).

These structures are mechanically stabilized by the action of pre-stress and are self-equilibrated without the application of an external force, i.e. their stability relies on the isometric straining of the inwardly pulling tensile members against the outwardly pushing compression members.

Because of the lack of physical connections between compression members, the pin-joints of a tensegrity structure have a predictable, linear response to any external loads over a wide range of different shapes, which can be very attractive, e.g. for deployable structures. Their light weight and aesthetic value add them special interest for the structural design.

Form-finding methods for tensegrity structures

The analysis of tensegrities, requires an initial procedure to find their self-stressed equilibrium configuration, that is not identical to that of the polyhedron usually taken as geometric basis. This procedure is known as *form-finding* and typically compute a critical parameter such as: a twisting angle, a cable-to-strut ratio or a force-to-length ratio, which is also known as the tension coefficient or the force density coefficient.

The existing form-finding methods are classified into two broad families: kinematical and statical methods.

The **kinematical methods** determine the geometry of a given tensegrity structure by maximizing the lengths of the struts while keeping constant the given lengths of the cables or, alternatively, by decreasing the length of the cables until a minimum is reached while the length of struts is kept constant.

The **statical methods** set up a relationship between equilibrium configurations of a tensegrity structure with given topology (i.e. a given number of nodes and connecting elements between them) and the forces in its members. This relationship can be analyzed by various methods: analytical solutions, force density method, energy method, ...

Design with XC. Model validation.

Tensegrities are structures that can have arbitrarily large displacements and rotations at the global level, for that reason a three-dimensional co-rotational truss is used for the analysis.

As the truss structure is loaded and deforms from each original configuration, each of its elements potentially does three things: it rotates, translates and deforms. A co-rotational formulation intends to separate the rigid body motions, integrated by rotation and translation, from strain producing deformations at the local element level. This is accomplished by attaching a local element coordinate system, which rotates and translates with the truss element. This co-rotating coordinate frame is oriented so that the x-axis is always directed along the axis of the truss element and, which respect to it, the rigid body rotations and translations are zero, only local strain producing deformations along the x-axis remains.

Calibration of the type of element The cables are modeled with tension-only corotational truss elements, for which the stiffness is removed if the element goes into compression.

The element is defined by two nodes, the cross-sectional area, an initial stress, and the elastic modulus of the material. It is possible to assign to the element an effective self weight, defined as the gravity component of weight per volume transverse to the cable.

The element condition at the beginning of the first step is determined from the initial stress, that must be greater than zero. The element is nonlinear and requires an iterative solution.

Table 1 shows the results obtained for the model depicted in figure 1 in different load conditions.

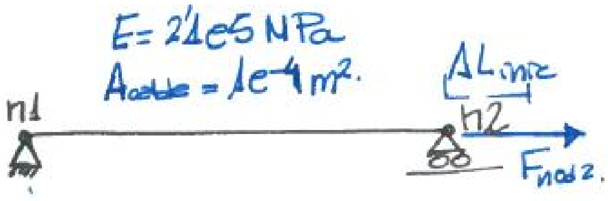

Figure 1: Model for element calibration

2D tensegriy. Snelson's X-frame Let us consider the two-dimensional tensegrity structure in Fig. 2, where the outside edges are cables and the diagonal are struts.

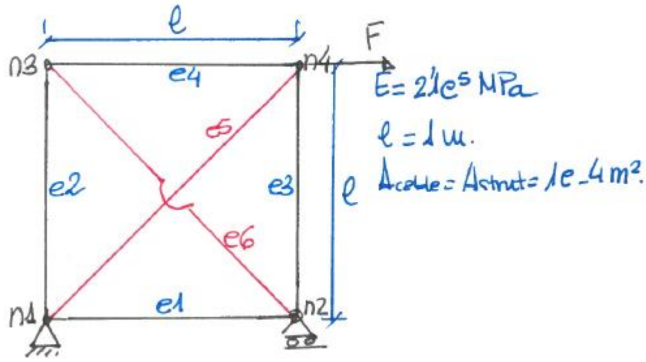

Figure 2: Snelson's X-frame model

In order to linearise the equilibrium equations, the force density method introduce for each element the force density:

$$q_{ij} = \frac{t_{ij}}{l_{ij}} \tag{1}$$

where t_{ij} is the axial force in cable/strut ij and l_{ij} is its length.

For the tensegrity structure in Fig. 2 a force density of 1 in each cable and -1 in each strut is a state of self-stress super stable, which is the strongest type of prestress stability. The analytical solution, in this case ($A_{cable} = A_{strut} = A$), gives:

$$F_{cables} = \sigma_{prestressing} \cdot A \cdot \frac{1}{1+\sqrt{2}} \tag{2}$$

$$F_{struts} = F_{cables} \cdot \sqrt{2} \tag{3}$$

$$\Delta L_{cables} = \frac{F_{cables} - \sigma_{prestressing} \cdot A}{A} \cdot \frac{l}{E} \tag{4}$$

$$\Delta L_{struts} = \frac{F_{struts}}{A} \cdot \frac{l \cdot \sqrt{2}}{E} \tag{5}$$

Figures 3 and 4 show the deformed shape and internal forces in a self-stressed configuration and when a horizontal point load is applied on the upper-right corner, respectively.

As can be seen in Table 2, the internal forces and deformations calculated with the finite element model in XC

Initial state	Load state		XC FE model results		Analytical results		Error	
σ_{prestr}	F_{nod2}	ΔL	σ_{elem}	ΔL	σ_{elem}	ΔL	ΔL	σ_{elem}
≈ 0	0	+0.001 m	210 MPa	+0.001 m	210 MPa	+0.001 m	0%	0 %
210 MPa	0	0	0	-0.001 m	0	-0.001 m	0%	0 %
210 MPa	21 kN	0	210 MPa	0	210 MPa	0	0%	0 %
210 MPa	0	+0.001 m	420 MPa	+0.001 m	420 MPa	+0.001 m	0%	0 %
210 MPa	0	-0.001 m	0 MPa	-0.001 m	0 MPa	-0.001 m	0%	0 %

Table 1: Results from the calibration of the element

coincide entirely with those forthcoming from the analytical solution.

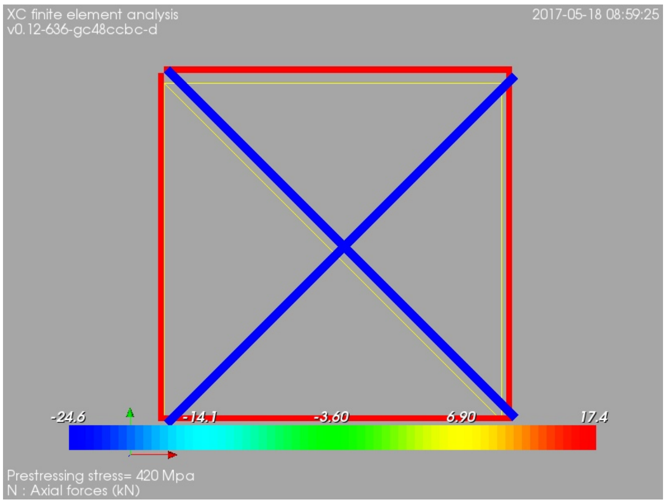

Figure 3: 2D tensegriy Snelson's X-frame. Axial internal forces in self-stressed equilibrium configuration [kN]

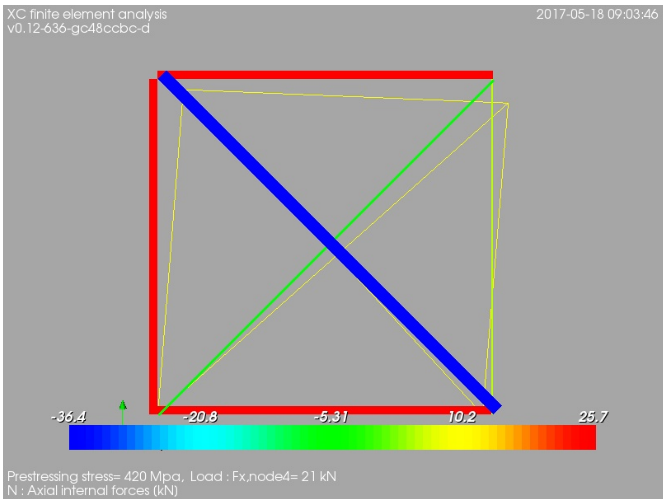

Figure 4: 2D tensegriy Snelson's X-frame. Axial internal forces when a horizontal 21 kN point load is applied on the upper-right corner [kN]

Cylindrical 3D tensegrities A cylindrical tensegrity structure, see Figs. 5 and 6, consists of two polygons, which are connected by a set of n bracing cables and n struts. Each polygon consists of $n \geq 3$ nodes interconnected by n cables. The n-plexes are defined by two parameters: the number j of steps between the nodes that are connected by a strut and the number k of steps between consecutively connected nodes along the convex hull of a given polygon. All the models we have analyzed here have the same connectivity $j = k = 1$.

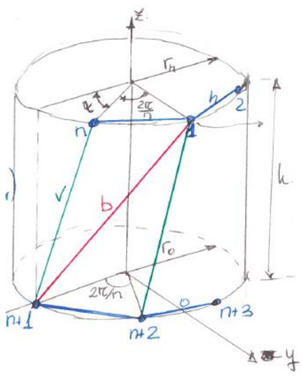

Figure 5: Cylindrical tensegrity

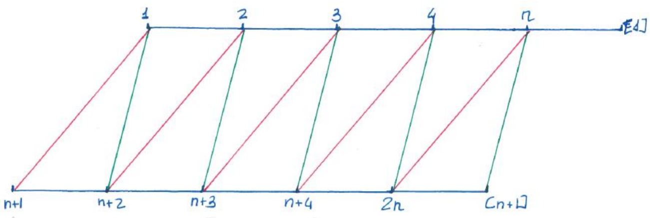

Figure 6: Unfolded cylindrical tensegrity

The form-finding of cylindrical tensegrities has been studied using several approaches, for instance, the dynamic relaxation procedure by Motro (2003) and nonlinear programming by Pellegrin. Using the force force densities $q = t/l$ in the cables and struts, and assuming that the top and bottom triangles lie in horizontal planes, vertical equilibrium gives:

$$q_{strut} = -q_{diag} \tag{6}$$

Super stability is attained for any set of positive cable forces densities that satisfy the condition:

$$q_{strut} = -2 \cdot \sin\left(\frac{\Pi}{n}\right) \cdot q_{sadd} \tag{7}$$

For the triplex model depicted in Fig. 7 a self-stressed configuration is analyzed (see Fig. 8) and a load case

	XC FE model results	Analytical results	Error
$\sigma_{prestr,cabl}$	420 MPa		
N_{cables}	17396.9696984 N	17396.9696197 N	≈ 0
N_{struts}	-24603.030197 N	-24603.0303803 N	≈ 0
ΔL_{cables}	-0.00117157286652 m	-0.00117157287525 m	≈ 0
ΔL_{struts}	-0.00165685423715 m	-0.00165685424949 m	≈ 0

Table 2: Results from the analysis of Snelson's X-frame

where three external point loads in -Z direction are applied on the nodes in the bottom face (see Fig. 9. The quadruplex, or 4-plex for short, is shown in Fig. 10. Finally, we show three examples (Figs. 11, 12 and 13) of tensegrity cylinders with a high degree of elements,such as the 20-plex, 50-plex, and 100-plex. The results of the XC model are compared with those from an analytical calculation and are found to be in very good agreement, as can be seen in Table 3.

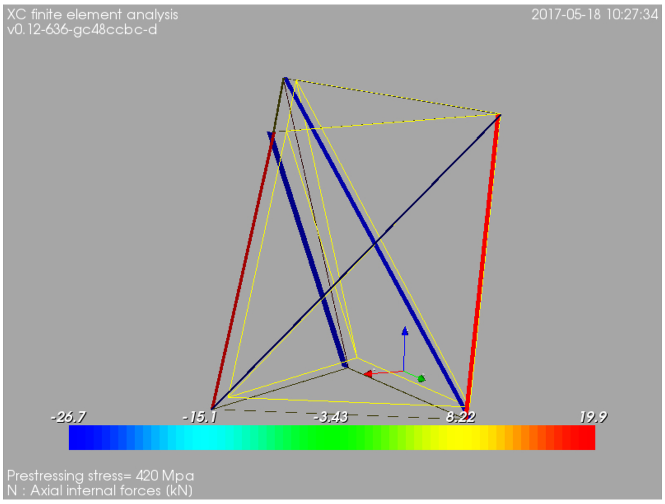

Figure 8: 3D 3-plex tensegriy prism. Axial internal forces in self-stressed equilibrium configuration [kN]

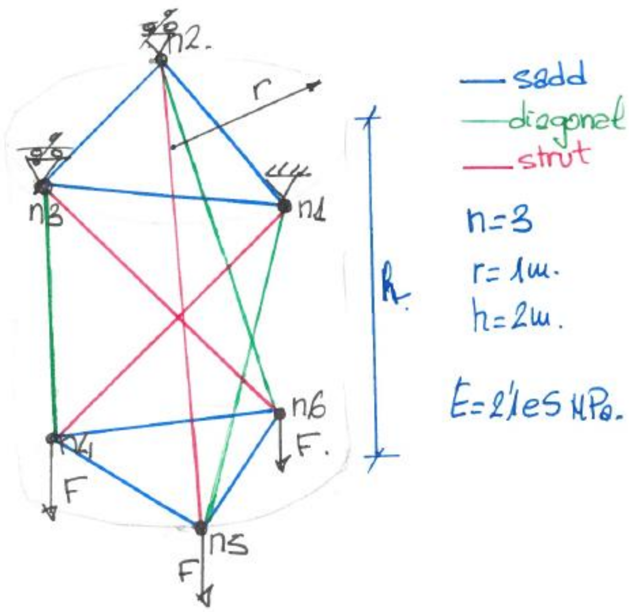

Figure 7: 3-plex tensegrity cylinder model

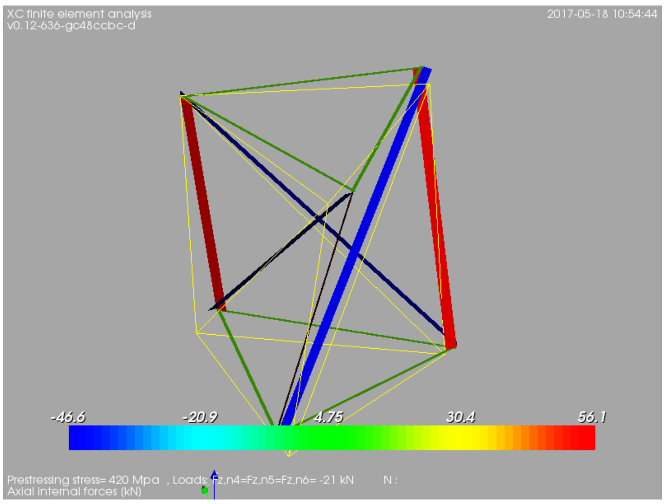

Figure 9: 3D 3-plex tensegriy prism. Axial internal forces when three 21 kN point loads in -Z direction are applied on the nodes in the bottom face [kN]

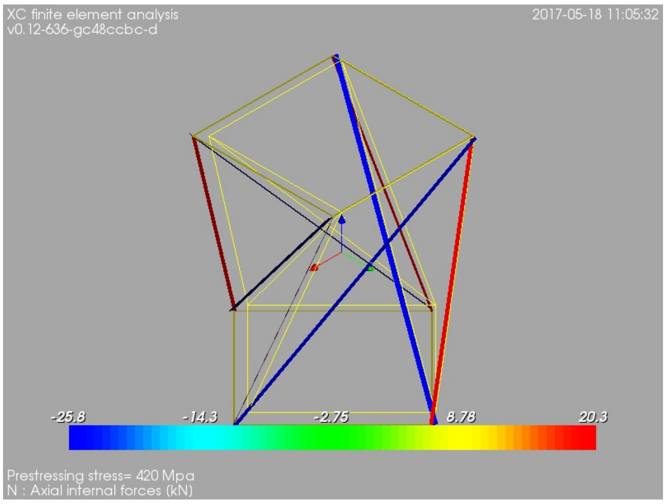

Figure 10: 3D 4-plex tensegriy prism. Axial internal forces in self-stressed equilibrium configuration [kN]

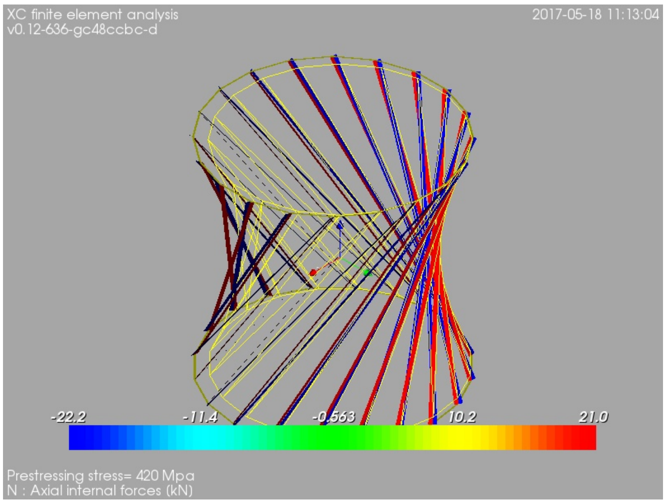

Figure 11: 3D 20-plex tensegriy prism. Axial internal forces in self-stressed equilibrium configuration [kN]

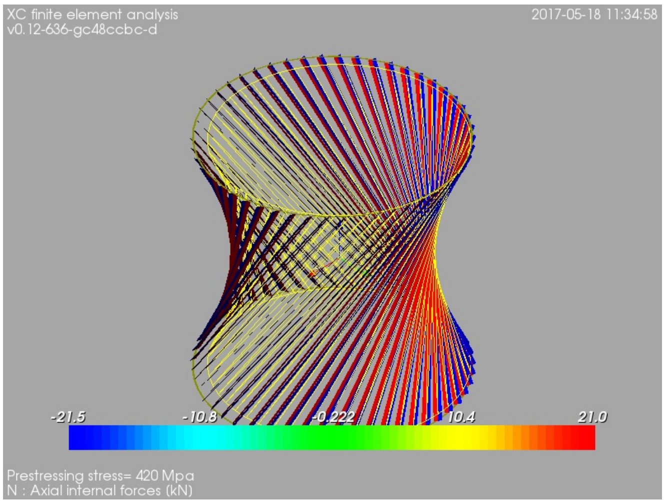

Figure 12: 3D 50-plex tensegriy prism. Axial internal forces in self-stressed equilibrium configuration [kN]

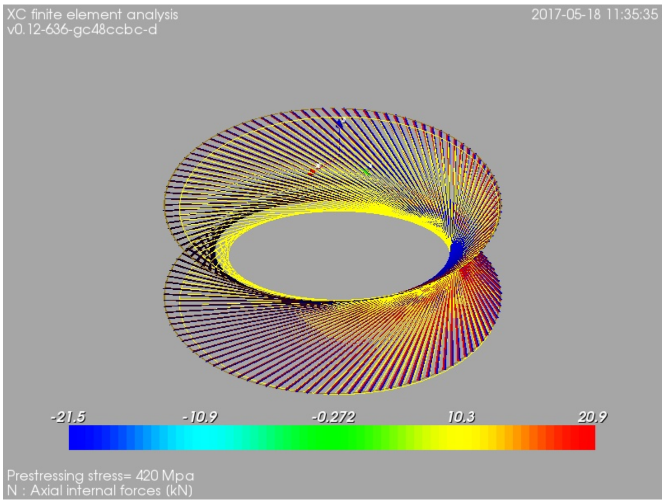

Figure 13: 3D 100-plex tensegriy prism. Axial internal forces in self-stressed equilibrium configuration [kN]

References

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[2] J.M. Puigoriol Forcada et al. Desarrollo de una metodolgía basada en el método de los elementos finitos para la proyección de estructuras tenségridas. *XVI Congreso Internacional de Ingeniería de Proyectos.*, pages 667–680, Valencia, 11-13 de julio de 2012.

[3] J.M. Mirats Tur M. Pagitz. Finite element based form-finding algorithm for tensegrity structures. *International Journal of Solids and Structures*, 46:3235–3240, 2009.

Prism	t_{strut} kN	l_{strut} m	q_{strut} kN/m	t_{diag} kN	l_{diag} m	q_{diag} kN/m	t_{sadd} kN	l_{sadd} m	q_{sadd} kN/m	check1	check2
3-plex	-26.7297	2.7771	-9.625	19.8633	2.0637	10.0	9.6101	1.7294	5.557	0.0	-0.0
4-plex	-25.8205	2.7196	-9.4944	20.3107	2.1392	9.0	9.4797	1.412	6.7135	0.0	0.0
20-plex	-22.1687	2.5099	-8.8326	21.0426	2.3824	9.0	8.8186	0.3124	28.2308	-0.0	-0.0
50-plex	-21.4755	2.4725	-8.6859	21.0311	2.4213	9.0	8.6721	0.1254	69.1655	0.0	-0.0
100-plex	-21.4566	4.7455	-4.5215	20.9125	4.6251	5.0	13.5462	0.1882	71.9739	0.0	-0.0
check1=$q_{strut} + q_{diag}$											
check2=$q_{strut} + 2 \cdot sin(\Pi/n) \cdot q_{sadd}$											

Table 3: Results from the analysis of cylindrical tensegrities