Implementation of a tension-stiffening model for the cracking nonlinear analysis of reinforced concrete elements in the finite element OSS XC

Ana Ortega[1], Luis C. Pérez Tato[2]

Abstract
The development of a smeared-crack model offers a general crack-modeling method that is independent of the structural configuration. It treats cracking as a constitutive material behaviour rather than a geometric discontinuity and lends itself well to implementation in large finite element codes. This paper deals with the implementation in XC of a constitutive model for reinforced concrete elements that takes into account the increase in stiffness of a cracked member due to the development of tensile stresses in the concrete between the cracks, effect known as tension-stiffening. The nonlinear analysis in XC of fiber-like sections with this constitutive model allows for a more general, direct and intuitive evaluation of the crack amplitude than applying the mostly specific formulae developed in the standards. The numerical results obtained by the program compare extremely well with existing designing results issued by other applied methods.

[1]*structural engineer - ana.ortega@ciccp.es*
[2]*structural engineer - l.pereztato@ciccp.es*

Contents

1. Introduction

The development of a smeared-crack model offers a general crack-modelling method that is independent of the structural configuration. It treats cracking as a constitutive material behaviour rather than a geometric discontinuity.

Quite a few models that modify the constitutive equation of steel or concrete after cracking have been proposed for nonlinear finite element analysis of reinforced concrete structures. The model that is to be implemented in XC, taken for reference [8], modifies the descending branch of the tensile stress-strain curve of concrete to take into account the tension-stiffening effect in an average way.

2. Crack control according to Eurocode-2, evaluation of the crack amplitude.

If a continuously increasing tension is applied to a tension member, the first crack will form when the tensile strength of the weakest section in the member is exceeded. The formation of this crack leads to a local redistribution of stresses within the section. At the crack, all the tensile force will be transferred to the reinforcement, and the stress in the concrete immediately adjacent to the crack must clearly be zero. With increasing distance from the crack, force is transferred by bond from the reinforcement to the concrete until, at some distance, l_t, from the crack, the stress distribution within the section remains unchanged from what it was before the crack formed. As further load is applied, a second crack will form at the next weakest section, though it will not form within l_t of the first crack since the stresses within this region will have been reduced by the formation of the first crack.

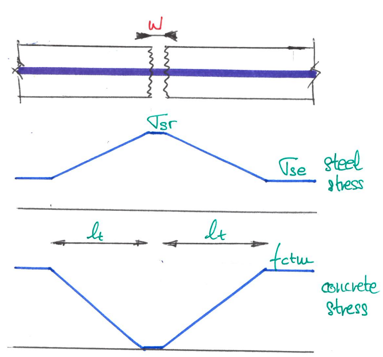


Figure 1. Definition of the crack width

As shown in figure 1 the crack width is the difference between the steel and concrete elongations over the length

$2l_t$, where l_t is the ■transmission length■ necessary to increase the concrete strength from 0 to the tensile strength f_{ctm}. Since no crack can form within l_t of an existing crack, this defines the minimum spacing of the cracks. The maximum spacing is $2l_t$, since if a spacing existed wider than this, a further crack could form.

The development of formulae for the prediction of crack widths given in clause 7.3.4 of EN 1992-1-1 follows from the description of the cracking phenomenon given above. If it is assumed that all the extension occurring when a crack forms is accommodated in that crack, then, when all the cracks have formed, the crack width will be given by the following relationship, which is simply a statement of compatibility:

$$w = S_{rm}\varepsilon_m$$

where w is crack width, S_{rm} is the average crack spacing and ε_m is the average strain. The average strain can be more rigorously stated to be equal to the strain in the reinforcement, taking account of tension stiffening, ε_{sm}, less the average strain in the concrete between cracks, ε_{cm} . Since, in design, it is a maximum width of crack which is required rather than the average, the final formula given in EN 1992-1-1 is

$$w_k = S_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})$$

In order to asses the mean strain in reinforcement taking into account the effects of tension stiffening, we have implemented in XC a constitutive model of concrete that modifies its tensile range so that the tension stiffening effect is considered in an average way.

3. Tension-stiffening concrete constitutive model

Stramandinoli and La Rovere have proposed ■An efficient tension-stiffening model for nonlinear analysis of reinforced concrete members■ [8]. The model uses an explicit formulation for the concrete stress–strain curve and thus can be easily implemented into a finite element code.

In the proposed model, concrete is assumed to behave like a linear-elastic material until its tensile strength is reached, so that a straight line defines initially the stress–strain curve, while in the post-cracking range, an exponential decay curve is adopted until yielding of reinforcement takes place. The exponential decay parameter (α) is a function of the member reinforcement ratio (ρ) and of the steel-to-concrete modular ratio ($\nu = E_s/E_c$), and is derived taking as basis the CEB tension-stiffening model.

This exponential decay curve is defined by the following equation:

$$\sigma_{ct} = f_{ct} e^{-\alpha(\frac{\varepsilon}{\varepsilon_{cr}})}$$

where,

f_{ct} is the concrete tensile strength;

ε_{cr} is the strain corresponding to the concrete tensile strength,

α is an exponential decay parameter.

The parameter α is derived by Stramandinoli and La Rovere taking as basis the CEB [1] model, and expressed as:

$$\alpha = 0.017 + 0.255(\nu\rho_{ef}) - 0.106(\nu\rho_{ef})^2 + 0.016(\nu\rho_{ef})^3$$

where,

ρ_{ef} is the member effective reinforcement ratio $\rho_{ef} = A_s/A_{c,ef}$

ν is the steel-to-concrete modular ratio $\nu = E_s/E_c$

4. Implementation of the model in XC

XC takes from OpenSees a material called ■concrete02■, implemented by Filip Filippou, that allows for concrete tensile strength.

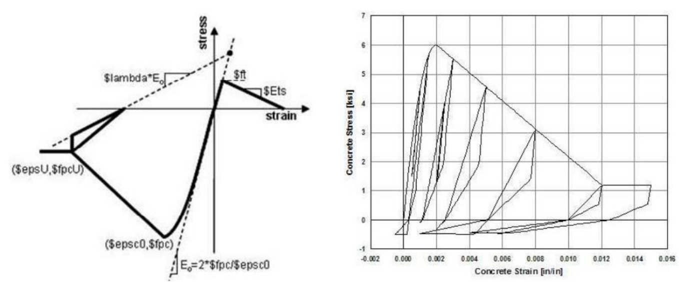


Figure 2. Concrete02 stress-strain relation and hysteresis behaviour (*OpenSees Manual,[4]*)

The stress-strain curve of this concrete and its typical hysteresis behaviour are illustrated in Fig. 2. The material behaviour in compression is defined by a maximum compressive strength f_{pc} for the strain ε_{c0} and the residual strength f_{pcu} achieved at the ultimate strain ε_{cu}; λ is the ratio between unloading slope at crushing strain and initial slope . The relation that describes the tensile behaviour is determined by the maximum tensile strength f_t and the slope coefficient that determines the decrease of the tensile strength E_{ts} .

To approach the exponential decay curve that characterises the post-cracking range in the model described in the section 3, a linear regression is calculated, so that we can easily introduce the law in the concrete02 definition (see Fig. 3)

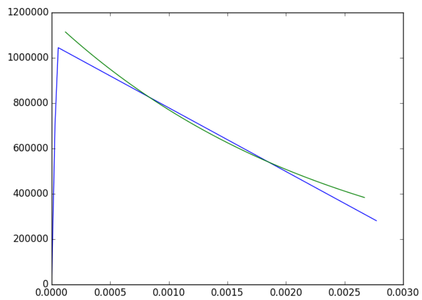


Figure 3. Tensile range of a concrete. Linear regression for approaching the exponential curve

XC also takes from OpenSees fibre models and non-linear solver algorithms. Fibre modelling of concrete and steel reinforcement allows bi-axial bending interaction with axial force acting at the same time. For the non-linear analysis, an iterative procedure with the loads applied in small increments is used. At each load increment step, direct iteration using the secant stiffness of the structure is employed. These capacities allow for a more general, direct and intuitive evaluation of the crack amplitude than applying the mostly specific formulae developed in the standards.

5. Verification

To verify the validity of the tension-stiffening model implemented in XC, several verification tests have been performed.

Firstly, two models are created on the basis of the corresponding pull-out experiments presented in the document of reference [8], p. 2074, the results of which are taken as reference values to be compared with those issued by XC.

Furthermore, some numerical examples conducting crack width calculation are taken from the reference [2], p. 7-8 to 7-14 , and performed in XC.

Pull-out tests The first test, V3, was conducted by Rostásy et al., *apud* Massicotte et al and modelled in a FE program by Stramandinoli and La Rovere [8]. It uses a bar of 6 m length and cross-section dimensions of $30cm \times 50cm$. A zero-length element, $30 \times 50cm^2$ in cross-section with a longitudinal steel ratio equal to 0.67%, made of the material depicted in table 1, is created in XC and subjected to tension in the axial direction.

Likewise, a second test conducted by Hwang and Riskalla, *apud* Gupta and Maestrini [9] is reproduced in XC. The cross-section is $17.8cm \times 30.5cm$, the longitudinal steel ratio is 1.476% and the material has the properties depicted in figure 2.

Figure 4 shows, for the two tests analysed, the stress (MPa) versus strain (‰) curves, obtained experimentally and numerically. It can be observed that XC models reproduce quite well the results published by Stramandinoli and La Rovere [8].

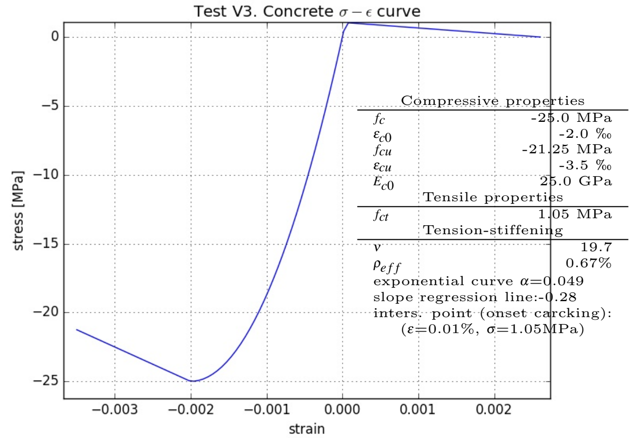


Table 1. Pull-out test V3 member, concrete material

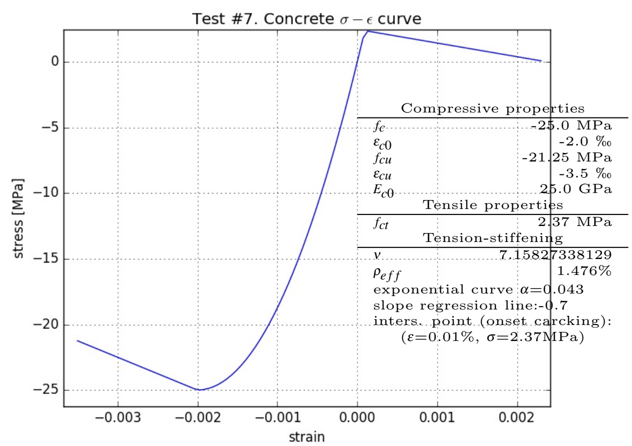


Table 2. Pull-out test # 7 member, concrete material

EC2 examples on evaluation of crack amplitude Some worked examples that carry out the evaluation of crack amplitude according to Eurocode 2, are extracted from the publication [2] and modelled in XC. The first one, example 7.3, is solved in the worked example following EC2 clause 7.3.4; the other three examples (7.5 a-b-c) obtain the crack amplitude by using the approximated method described in EC2 clause 7.4.

6. Conclusions. Further work

The material constitutive model implemented in **XC** approaches very well the numerical analysis and the

EC2 worked example	RC section charact. *(see table)*	Concrete material *(see table)*	w_k EC2ex (mm)	XC results									
				h_{nf} (m)	$\varepsilon_{c,min}$ (‰)	$\sigma_{c,min}$ (MPa)	$\varepsilon_{s,m}$ (‰)	$\sigma_{s,m}$ (MPa)	$h_{c,eff}$ (m)	$\rho_{s,eff}$	$s_{r,max}$ (m)	ε_{cm} (‰)	w_k (mm)
7.3	6	4	0.184	-0.211	-0.53	-13.76	0.91	181.96	0.13	0.052	0.214	0.05	0.184
7.5-a	7	5	0.306	-0.164	-0.66	0.0	1.2	239.17	0.112	0.047	0.256	0.05	0.294
7.5-b	8	5	0.213	-0.178	-0.61	0.93	0.95	189.81	0.107	0.064	0.233	0.05	0.211
7.5-c	9	5	0.12	-0.205	-0.52	-15.0	0.63	125.6	0.098	0.114	0.206	0.05	0.12

h_{nf}: neutral fibre depth
$\varepsilon_{c,min}$: minimum strain in concrete fibres
$\sigma_{c,min}$: minimum stress in concrete fibres
$\varepsilon_{s,m}$:mean strain in reinforcement taking into account the effects of tension stiffening
$\sigma_{s,m}$: mean stress in reinforcement taking into account the effects of tension stiffening
$h_{c,eff}$: depth of the effective area
$\rho_{s,eff}$: effective reinforcement ratio
$s_{r,max}$: maximum crack spacing
w_k: crack width

Table 3. Comparison between XC results and worked examples from [2]

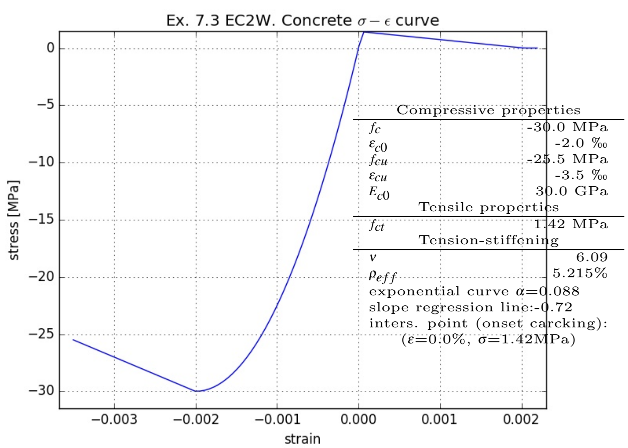


Table 4. Test example 7.3 [2], concrete material

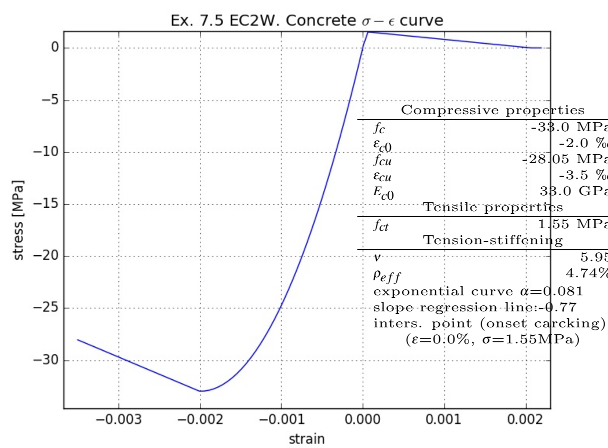
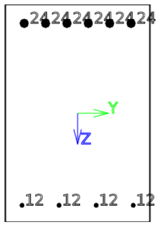


Table 5. Tests example 7.5 [2], concrete material

example_7.3_EC2W

Test example 7.3 EC2 Worked examples. Section definition



width: $b = 0.40\ m$
depth: $h = 0.60\ m$

Materials - mechanical properties:

Concrete: C30-37	Modulus of elasticity: $E_c = 32.84\ GPa$
Steel: S450C	Modulus of elasticity: $E_s = 200.00\ GPa$

Sections - geometric and mechanical characteristics:

Gross section:

$A_{gross} = 0.240\ m^2$ Inertia tensor (cm^4): $\begin{pmatrix} 75.26 & 0.00 & 0.00 \\ 0.00 & 72.00 & 0.00 \\ 0.00 & 0.00 & 32.00 \end{pmatrix}$

C.O.G.: $(0.000, 0.000)\ m$

Homogenized section:

$A_{homog.} = 0.261\ m^2$ Inertia tensor (cm^4): $\begin{pmatrix} 75.26 & 0.00 & 0.00 \\ 0.00 & 84.55 & 0.00 \\ 0.00 & 0.00 & 34.25 \end{pmatrix}$

C.O.G.: $(0.000, -0.014)\ m$

Passive reinforcement:

Total area $A_s = 31.67\ cm^2$ Geometric quantity $\rho = 13.19‰$

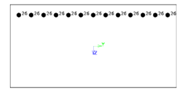
Layers of main reinforcement:

Id	N^o bars	ϕ (mm)	area (cm^2)	c. geom. (‰)	eff. cover (cm)	y_{COG} (m)	z_{COG} (m)
	6	0.0	4.52	1.88	5.2	0.000	-0.248
	4	0.0	1.13	0.47	4.6	0.000	0.254

Table 6. Test example 7.3 EC2 Worked examples. Section definition (example_7.3_EC2W).

example_7.5_EC2W_0.2mm

Test example 7.5 EC2 Worked examples - $w_k \approx 0.2mm$. Section definition



width: $b = 1.00\ m$
depth: $h = 0.50\ m$

Materials - mechanical properties:

Concrete: C33	Modulus of elasticity: $E_c = 33.59\ GPa$
Steel: S450C	Modulus of elasticity: $E_s = 200.00\ GPa$

Sections - geometric and mechanical characteristics:

Gross section:

$A_{gross} = 0.500\ m^2$ Inertia tensor (cm^4): $\begin{pmatrix} 2290.00 & 0.00 & 0.00 \\ 0.00 & 104.17 & 0.00 \\ 0.00 & 0.00 & 416.67 \end{pmatrix}$

C.O.G.: $(0.000, 0.000)\ m$

Homogenized section:

$A_{homog.} = 0.542\ m^2$ Inertia tensor (cm^4): $\begin{pmatrix} 2290.00 & 0.00 & 0.00 \\ 0.00 & 117.68 & 0.00 \\ 0.00 & 0.00 & 449.19 \end{pmatrix}$

C.O.G.: $(-0.000, -0.014)\ m$

Passive reinforcement:

Total area $A_s = 69.02\ cm^2$ Geometric quantity $\rho = 13.80‰$

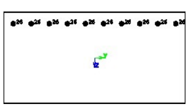
Layers of main reinforcement:

Id	N^o bars	ϕ (mm)	area (cm^2)	c. geom. (‰)	eff. cover (cm)	y_{COG} (m)	z_{COG} (m)
	13	0.0	5.31	1.06	5.3	-0.000	-0.187

Table 8. Test example 7.5 EC2 Worked examples - $w_k \approx 0.2mm$. Section definition (example_7.5_EC2W_0.2mm).

example_7.5_EC2W_0.3mm

Test example 7.5 EC2 Worked examples - $w_k \approx 0.3mm$. Section definition



width: $b = 1.00\ m$
depth: $h = 0.50\ m$

Materials - mechanical properties:

Concrete: C33	Modulus of elasticity: $E_c = 33.59\ GPa$
Steel: S450C	Modulus of elasticity: $E_s = 200.00\ GPa$

Sections - geometric and mechanical characteristics:

Gross section:

$A_{gross} = 0.500\ m^2$ Inertia tensor (cm^4): $\begin{pmatrix} 2290.00 & 0.00 & 0.00 \\ 0.00 & 104.17 & 0.00 \\ 0.00 & 0.00 & 416.67 \end{pmatrix}$

C.O.G.: $(0.000, 0.000)\ m$

Homogenized section:

$A_{homog.} = 0.532\ m^2$ Inertia tensor (cm^4): $\begin{pmatrix} 2290.00 & 0.00 & 0.00 \\ 0.00 & 114.75 & 0.00 \\ 0.00 & 0.00 & 442.87 \end{pmatrix}$

C.O.G.: $(-0.000, -0.011)\ m$

Passive reinforcement:

Total area $A_s = 53.09\ cm^2$ Geometric quantity $\rho = 10.62‰$

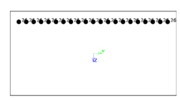
Layers of main reinforcement:

Id	N^o bars	ϕ (mm)	area (cm^2)	c. geom. (‰)	eff. cover (cm)	y_{COG} (m)	z_{COG} (m)
	10	0.0	5.31	1.06	5.3	-0.000	-0.187

Table 7. Test example 7.5 EC2 Worked examples - $w_k \approx 0.3mm$. Section definition (example_7.5_EC2W_0.3mm).

example_7.5_EC2W_0.1mm

Test example 7.5 EC2 Worked examples - $w_k \approx 0.1mm$. Section definition



width: $b = 1.00\ m$
depth: $h = 0.50\ m$

Materials - mechanical properties:

Concrete: C33	Modulus of elasticity: $E_c = 33.59\ GPa$
Steel: S450C	Modulus of elasticity: $E_s = 200.00\ GPa$

Sections - geometric and mechanical characteristics:

Gross section:

$A_{gross} = 0.500\ m^2$ Inertia tensor (cm^4): $\begin{pmatrix} 2290.00 & 0.00 & 0.00 \\ 0.00 & 104.17 & 0.00 \\ 0.00 & 0.00 & 416.67 \end{pmatrix}$

C.O.G.: $(0.000, 0.000)\ m$

Homogenized section:

$A_{homog.} = 0.568\ m^2$ Inertia tensor (cm^4): $\begin{pmatrix} 2290.00 & 0.00 & 0.00 \\ 0.00 & 125.01 & -0.00 \\ 0.00 & -0.00 & 466.20 \end{pmatrix}$

C.O.G.: $(0.000, -0.022)\ m$

Passive reinforcement:

Total area $A_s = 111.50\ cm^2$ Geometric quantity $\rho = 22.30‰$

Layers of main reinforcement:

Id	N^o bars	ϕ (mm)	area (cm^2)	c. geom. (‰)	eff. cover (cm)	y_{COG} (m)	z_{COG} (m)
	21	0.0	5.31	1.06	5.3	0.000	-0.187

Table 9. Test example 7.5 EC2 Worked examples - $w_k \approx 0.1mm$. Section definition (example_7.5_EC2W_0.1mm).

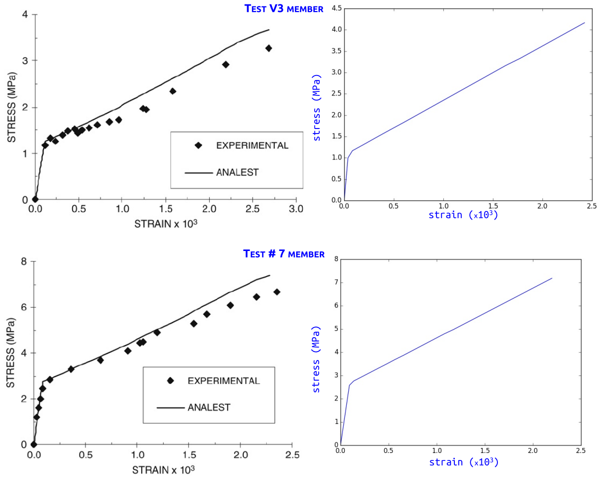


Figure 4. Results pull-out tests. Comparison between numerical-experimental results published in [8] (left) and results obtained with **XC** (right)

experimental curves obtained in the pull-out tests with which has been compared, as shown in Fig. 4 An excellent agreement can be observed from Table 3 by comparison between the crack amplitude calculated for four worked examples of Eurocode 2 in the publication [2] and the corresponding crack widths obtained by the XC program.

The test runs and the comparison with existing designing results for different cases show that the program delivers results corresponding to the up to now applied methods for evaluation of crack amplitude. One can actually, as a further work, apply the program for all the elements in a structure and for all design actions and combinations relating to the cracking limit state.

Here, hosted in the platform GitHub, you can find the **XC** source-code and the tests referred to in this article:

XC source-code.
test_concrete02_02.py
test_smearedCracking_01.py
test_smearedCracking_02.py
test_smearedCracking_03.py
test_smearedCracking_04.py
test_smearedCracking_05.py
test_smearedCracking_06.py

References

[1] Ceb-fip model code 90. Technical report, 1993.

[2] European Concrete Platform ASBL. Eurocode 2, worked examples. Technical report, 2008.

[3] Comisión Permanente del Hormigón. Instrucción de hormigón estructural (ehe-08). Technical report, Ministerio de Fomento, 2008.

[4] Silvia Mazzoni Frank McKenna Michael H. Scott Gregory L. Fenves et al. Opensees command language manual. Technical report, Earthquake Engineering Research Center. College of Engineering. University of California, Berkeley, 2006.

[5] F.Biasioli G.Mancini M.Just M.Curbach J.Walraven S.Gmainer J.Arrieta R.Frank C.Morin F.Robert. Eurocode 2: background & applications. design of concrete buildings. Technical report, European Commission, 2014.

[6] Elwi A.E. MacGregor J.G. Massicote B. Tension-stiffening model for planar reinforced concrete members. *ASCE Journal of Structural Engineering*, 106(11):3039–3058, 1990.

[7] EA. W. Beeby R. S. Narayanan. Designers' guide to en1992-1-1 and en1992-1-2. Technical report, 2009.

[8] Renata S.B. Stramandinoli Henriette L. La Rovere. An efficient tension-stiffening model for nonlinear analysis of reinforced concrete members. *Engineering Structures*, 30:2069–2080, 2008.

[9] Gupta A. Maestrini S.R. Tension-stiffness model for reinforced concrete bars. *ASCE Journal of Structural Engineering*, 106(3):769–791, 1990.